# General and particular solution of a first-order differential equation

Given the following differential equation:

$$x\frac{dy}{dx} - y = xe^{x/y}$$

1. Determine the general solution of the differential equation.
2. Determine the particular solution of the differential equation if $y(1) = 3$.

## Solution

1. Rewrite:

$$x\,dy = (xe^{x/y} + y)\,dx$$

These are homogeneous of degree 1. We divide by $x^1$:

$$dy = \left(e^{x/y} + \frac{y}{x}\right)dx$$

We set $v = y/x$. Then $y = vx$, $dy = v\,dx + x\,dv$. Substituting:

$$v\,dx + x\,dv = (e^v + v)\,dx$$

$$\frac{1}{e^v}\,dv = \frac{1}{x}\,dx$$

Solving:

$$-e^{-v} = \ln(x) + C$$

$$-e^{-y/x} = \ln(x) + C$$

2. Using the initial conditions:

$$-e^{-3/1} = 0 + C$$

$$C = -e^{-3}$$

Therefore:

$$-e^{-y/x} = \ln(x) - e^{-3}$$